

Effects of temperature distribution and level on heat transfer on a rotating free disk

ZHAO Xi, XU Guo-qiang, LUO Xiang

(National Key Laboratory of Science and Technology on Aero-Engine Aero-thermodynamics,
School of Jet Propulsion, Beijing University of Aeronautics and Astronautics,
Beijing 100191, China)

Abstract: In gas turbine engines, with the existence of the intense forced convection and significant buoyancy effects, temperature distribution and level on turbine or compressor disks affect the heat transfer characteristics strongly. In this paper, numerical simulations were performed to analyze these influences for a free disk, with the laminar and turbulent flow respectively. The influences of temperature distribution on the heat transfer were observed by using incompressible cooling air, and temperature profiles of n th order monomial and polynomial were assumed on the disk. The analysis revealed that the heat transfer for two flow states on the free disk is determined by the exponent n of the monomial profile when specifying the rotating Reynolds number; for an arbitrary polynomial profile, the local Nusselt number can be deduced from results of monomial profiles. To study the effects of temperature level on heat transfer singly, monomial profiles were used and the local Nusselt number of compressible and incompressible cooling air were compared. And both for two flow states, the following conclusions could be drawn: the relative difference of local Nusselt number is mainly controlled by nondimensional local temperature difference, and almost independent of the monomial's coefficient C , exponent n and the rotating Reynolds number. Subsequently, a correction method for heat transfer of the free disk is presented and verified computationally, with which the local Nusselt number, obtained with a uniform and low temperature profile, can be revised by arbitrary distribution and high temperature magnitude.

Key words: thermal boundary condition; temperature boundary condition; rotating disk; free disk; heat transfer

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Nomenclature

a	coefficient of monomial or polynomial	z	axial coordinate
c_p	specific heat at constant pressure of air	ϵ_r	relative difference of local Nusselt number ($\epsilon_r = (Nu_{r,incp} - Nu_{r,cp}) / Nu_{r,incp}$)
$f(\theta_r)$	function of ϵ_r with θ_r	θ	nondimensional max temperature difference ($\theta = (T_{w,R} - T_i) / T^*$)
h	heat transfer coefficient	θ_r	nondimensional local temperature difference ($\theta_r = (T_{w,r} - T_i) / T^*$)
n	exponent of monomial or polynomial	λ	thermal conductivity of air
Nu_r	local Nusselt number ($Nu_r = h_r r / \lambda$)	μ	dynamic viscosity of air
Pr	Prandtl number	ρ	density of air
q	heat flux	$\sigma_{r,n}$	ratio of Nusselt number ($\sigma_{r,n} = Nu_{r,n} / Nu_{r,0}$)
r	radial coordinate	ω	rotating velocity
R	max radius of the free disk		
Re_ω	rotating Reynolds number ($\rho \omega r^2 / \mu$)		
T	temperature		
T^*	reference temperature (300 K)		
C	coefficient		

Subscripts

c	critical point from laminar to turbulent	incp	value with incompressible cooling air
cp	value with compressible cooling air	n	with reference to exponent n
exp	experimental value	r	with reference local radial coordinate
f	value at opening boundary	R	value at the edge of free disk

Introduction

Some assumptions and simplifications are usually made during the design and experimental studies of flow and heat transfer for gas turbine engines. For instance especially, the neglect of the effects of thermal boundary conditions is a conventional assumption. However, with the presence of large mass flow rate and high rotating speed in turbine or compressor disk cavities^[1], the forced convection and the buoyancy effects are so significant that the thermal boundary conditions, particularly the temperature distribution and level on turbine disks, affect the heat transfer strongly.

It is researched extensively that the influence of some simple temperature distribution with a trend with uniform, step, ramp and parabola temperature profiles on heat transfer coefficient on flat plate, cylinder, and turbine blades, and the reader is referred to Ref. [2-6]. For the effects of temperature distribution on rotating disks in aero-engines, researchers put focus on a simplified rotating free disk. Dorfman solved the governing equation of flow and heat transfer around a free disk with simple monomial temperature boundary condition, and accurate theoretical solutions of local Nusselt number were obtained with the incompressible flow assumption^[7-8]. As a complement to Dorfman's work, Owen and Rogers solved the integral energy equation and presented an approximate result of laminar heat transfer with a bigger range of monomial distribution and Prandtl number^[9]. In order to find the relationship of heat transfer with arbitrary temperature profiles on the free disk in laminar flow, LÜ et al. performed numerical investigations and provided a practicable conclusion^[10].

For the effects of temperature magnitude on heat transfer in turbine engines, many researchers mainly investigated the mechanism of the buoyancy which yields from the temperature difference and the centrifugal force^[11-15]. For a free disk with laminar flow, ZHAO et al. studied the variation of temperature level with local Nusselt number by comparing the compressible cooling air with the incompressible, and they found that the deviation of local Nusselt number is dominantly determined by the local temperature difference between the disk surface and the main flow^[16].

In this paper, numerical simulation was performed to study the effects of temperature distribution and magnitude on heat transfer on a free disk. Universal temperature profiles were applied and the integral influences of temperature level on governing equations, hence on the heat transfer, were discussed both for the laminar and turbulent flow. In this consideration, a deep insight into the influence of temperature boundary conditions on the heat transfer for real turbine or compressor disks would be presented.

1 Description of numerical methods

1.1 Theoretical solution of heat transfer of free disk

For the sake of the theoretic generalization of engineering applications, a rotating free disk (illustrated in Fig. 1), the simplest configuration of turbine disk which has classic analytical solution of flow and heat transfer, was used in this paper. To get the heat transfer coefficients on the free disk, Dorfman^[8] and Owen et al.^[9], solved the integral energy equation respectively with static, incompressible, and constant physical properties assumptions. For laminar flow, when the Prandtl number $Pr=0.71$ and the tem-

perature on the free disk is uniform (i. e. $T_w - T_f = Cr^n$, $n=0$, where T_w is the local temperature on the surface of the free disk, T_f is the temperature of main flow, and C is the coefficient), Dorfman provided that the local Nusselt number Nu_r is given by

$$Nu_r = 0.326\sqrt{Re_{\omega,r}} \quad (1)$$

where $Nu_r = h_r r / \lambda$, r is the local radius of the free disk, λ is the thermal conductivity of cooling air, $h_r = q_w / (T_w - T_f)$ is the local heat transfer coefficient, q_w is local heat flux on the free disk and $Re_{\omega,r} = \rho \omega r^2 / \mu$ is the rotating Reynolds number. When $T_w - T_f = Cr^2$, then

$$Nu_r = 0.518\sqrt{Re_{\omega,r}} \quad (2)$$

When temperature distribution on the disk is an n th order monomial $T_w - T_f = Cr^n$ and the Prandtl number Pr is arbitrary, Owen et al. provided that:

$$Nu_r = 0.308(n+2)^{0.5} Pr^{0.5} (Re_{\omega,r})^{0.5} \quad (3)$$

It should be noticed that Eq. (1) and Eq. (2) are accurate results^[8], and Eq. (3) is a fitted formula^[9] with an error of $\pm 12.6\%$ when $0.71 < Pr < 2$, $0 < n < 3$. For heat transfer on the free disk with turbulent flow, Dorfman solved the integral energy equation basing on the velocity field, worked out by applying logarithm law to the boundary layer, and obtained that:

$$Nu_r = 0.0197(n+2.6)^{0.2} Re_{\omega,r}^{0.8} Pr^{0.6} \quad (4)$$

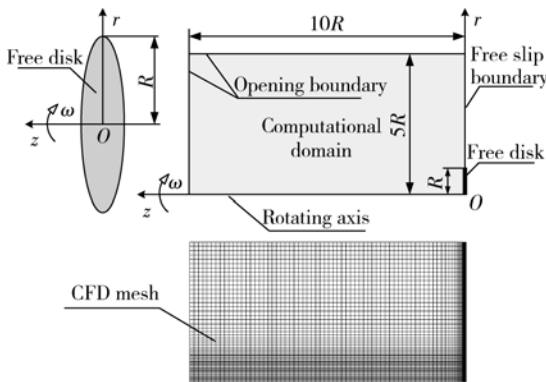


Fig. 1 Schematic diagram of computational domain and mesh of free disk

In above equations, the effect of thermal boundary condition on the heat transfer of the

free disk is exhibited exclusively by the exponent n rather than the coefficient C . And by comparing the heat transfer results of the two flow state with the incompressible assumption, it is ready to draw that the local heat transfer coefficient on the free disk is a constant with the given rotating speed and Pr for the laminar flow rather than the turbulent flow.

1.2 Numerical method

In a numerical simulation, this paper used the FLUENT codes to take up with the effects of temperature distribution and level on the heat transfer on a rotating free disk. The numerical settings were similar to the works of LÜ et al.^[10], and ZHAO et al.^[16] and a brief description are as follows.

Fig. 1 shows the computational fluid dynamics (CFD) model and mesh schematically. The free disk, with radius $R=0.2$ m, rotates in huge space in z axis. The computational domain is simplified to two-dimensional (2-D) due to the axial symmetry, with range of $0 \leq z \leq 10R$ for the axial direction and $0 \leq r \leq 5R$ for the radial direction. For the big gradient of velocity and temperature, the mesh near the surface of the free disk is refined. The mesh number ($r \times z$) is about 190×200 for laminar flow and 210×230 for turbulent flow.

The free disk, which has a critical rotating Reynolds number $Re_{\omega,c} = 2 \times 10^5$ from laminar flow to turbulent flow^[11], rotates around z axis with $Re_{\omega} \approx 0.813 \times 10^5$, 1.355×10^5 , 1.898×10^5 for laminar and with $Re_{\omega} \approx 4.067 \times 10^5$, 6.779×10^5 , 9.491×10^5 for turbulence; monomial and polynomial temperature profile are applied on the disk respectively as:

$$T_w - T_f = Cr^n \quad (5)$$

$$T_w - T_f = a_0 + a_1 r + a_2 r^2 + \dots + a_n r^n \quad (6)$$

where C , a_0 , a_1 , a_2 , $\dots$, a_n are constants and $n=0, 1, 2, 3$ in this paper. The free slip boundary is free slip and adiabatic. The operating pressure is 101325 Pa. At the opening boundary, the flow direction and other parameters are unknown; however, its boundary type in FLUENT

is set as outlet with considering there are back-flow, then the gauge pressure and the backflow temperature is assigned as $p_i=0\text{Pa}$ and $T_i=300\text{K}$ respectively.

The fluid used in this simulation was incompressible and compressible air respectively. For the incompressible case, the physical properties of the air was determined from the temperature of the opening boundary T_i , then the air density, thermal conductivity, dynamic viscosity, and specific heat capacity are as follows: $\rho=1.185\text{kg/m}^3$, $\lambda=0.0261\text{W/(m}\cdot\text{K)}$, $\mu=1.831\times 10^{-5}\text{Pa}\cdot\text{s}$, $c_p=1004.4\text{J/(kg}\cdot\text{K)}$. For the compressible case, the air density was solved by the idea gas law for compressible flows with the reference temperature T_i , and other physical properties are the same as that in compressible case. However, according to Dorfman^[8], when the rotating speed is relatively low, the variation of pressure near the disk is negligible, and then the density of the cooling air is mainly determined by its temperature rather than its pressure.

Static governing equations^[17] of flow and heat transfer in stationary reference frame were solved. The pressure work and the viscous dissipation term were neglected. The advection scheme was 2th order upwind and the pressure-velocity coupling scheme was semi-implicit method for pressure linked equation (SIMPLE) arithmetic. For the turbulence, the turbulent model was realizable $k-\epsilon$, and enhanced wall function was used as the near wall treatment.

1.3 Comparison of numerical results with theoretical solutions

With the assumption of incompressible and constant physical properties, Fig. 2 and Fig. 3 show the comparison of numerical results of local Nusselt number Nu_r with the analytical solutions for laminar and turbulent flow respectively. In Fig. 2, when $n=0$ and $n=2$, the numerical data of Nu_r agree with Dorfman's precise laminar solution very well (the max error is about $\pm 1\%$), as shown in Eqs. (1) and (2);

When $n=1$ and $n=3$, the difference between the CFD results and Owen's data in Eq. (3) is a little bigger (the max value is about $\pm 5\%$). Considering Eq. (3) is not an accurate solution, so this difference could be neglected. In Fig. 3, for the turbulent flow, the numerical predictions consist with the Dorfman's solution in Eq. (4) commendably. Through the comparison above, the validity of the CFD code has been substantiated.

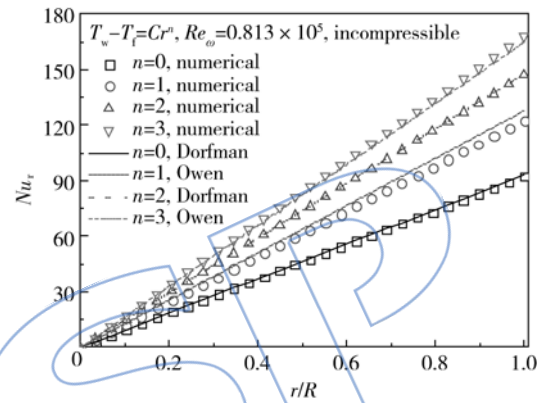


Fig. 2 Comparison of numerical and theoretical Nu_r for laminar flow

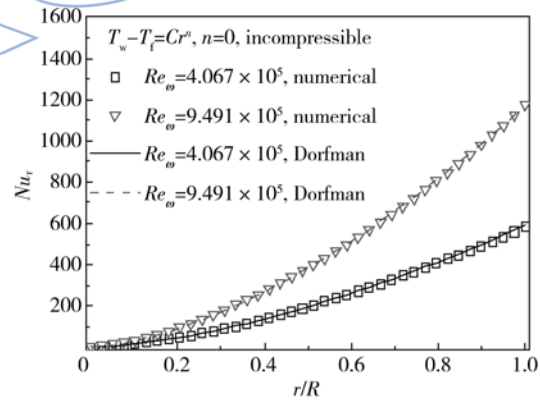


Fig. 3 Comparison of numerical and theoretical Nu_r for turbulent flow

2 Effects of temperature distribution

To study the influence of temperature distribution on the heat transfer separately, the assumption of incompressible flow was used, and both monomial and polynomial temperature profiles on the free disk were performed. For laminar flow, a detailed review of this problem was presented by LÜ et al.^[10]; however, it has also

been cited here as a comparison with the results of turbulent flow.

2.1 Monomial temperature profiles

When monomial temperature profiles were applied on the disk (see Eq.(5)), Fig.4 and Fig.5 show the effects of coefficient C on the local Nusselt number for laminar and turbulent flow respectively. It is obvious to see that Nu_r does not change with C , and this condition is consistent with the theory solution.

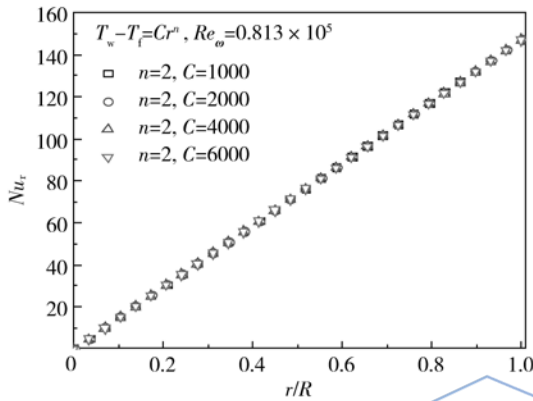


Fig.4 Effect of coefficient C on Nu_r for incompressible and laminar flow with monomial temperature distribution

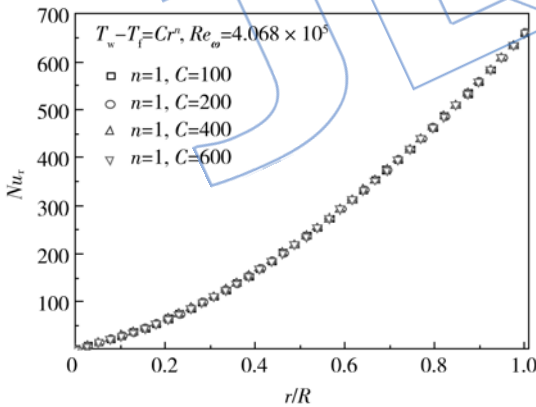


Fig.5 Effect of coefficient C on Nu_r for incompressible and turbulent flow with monomial temperature distribution

Fig. 6 and Fig. 7 illustrate the influence of the exponent n on Nu_r for the two flow state. In the charts, Nu_r is increased with n rising, for example, with laminar flow, Nu_r for $n=1$, $n=2$ and $n=3$ is increased than that for $n=0$ by 32%, 59% and 81% respectively. This is caused

by the higher temperature gradient on the disk along the main flow direction with a bigger n .

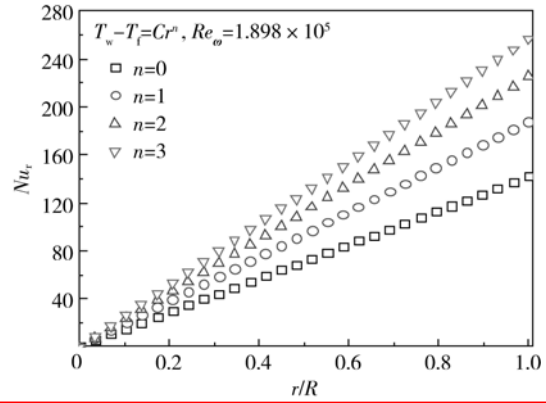


Fig.6 Effect of exponent n on Nu_r for incompressible and laminar flow with monomial temperature distribution

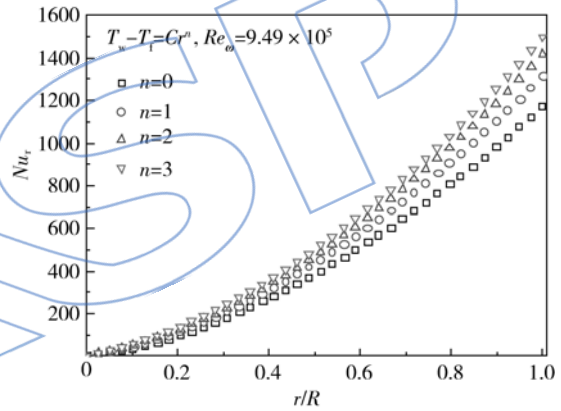


Fig.7 Effect of exponent n on Nu_r for incompressible and turbulent flow with monomial temperature distribution

2.2 Polynomial temperature profiles

When polynomial temperature profiles were performed, as Eq.(6), the heat transfer results of the free disk can be deduced by applying the calculating method to problems of boundary layer flow provided by Eckert et al.^[18]. Say $Nu_{r,0}$, $Nu_{r,1}$, $Nu_{r,2}$, $\dots$, $Nu_{r,n}$ are the local Nusselt number for $n=0, 1, 2, \dots, n$ when applying monomial temperature distribution (see Eq.(5)), and the ratio of the local Nusselt number is defined as follow:

$$\sigma_{r,n} = \frac{Nu_{r,n}}{Nu_{r,0}} \quad (7)$$

Then the local Nusselt number for an arbitrary polynomial temperature profile on the free disk

is obtained by:

$$Nu_r = \left(\frac{a_0 + a_1 r \sigma_{r,1} + a_2 r^2 \sigma_{r,2} + \dots + a_n r^n \sigma_{r,n}}{a_0 + a_1 r + a_2 r^2 + \dots + a_n r^n} \right) \quad (8)$$

To check the validity of Eq. (8), CFD simulations were conducted both for monomial and polynomial temperature profile. Fig. 8 and Fig. 9 plot the comparison of the numerical predictions of local heat transfer with the results computed from Eq. (8) for laminar and turbulent flow respectively. The good agreement of Nu_r indicates that Eq. (8) is quite accurate. According to Eq. (8), the effects of coefficients and exponent of the polynomial on the heat transfer can be determined.

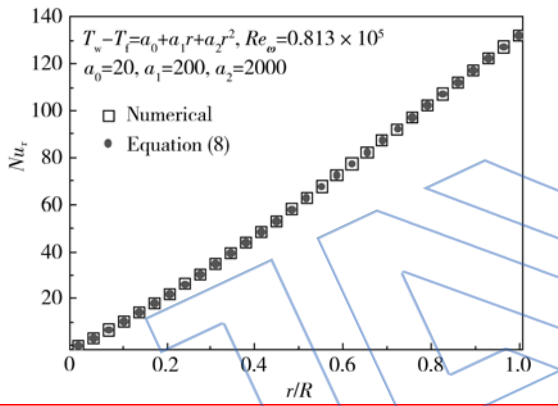


Fig. 8 Comparison of Nu_r calculated by CFD and Eq. (8) for laminar flow

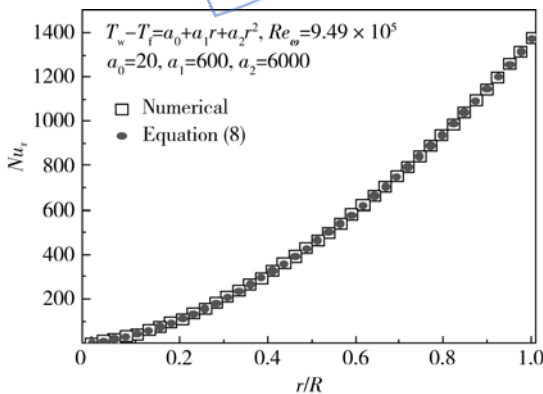


Fig. 9 Comparison of Nu_r calculated by CFD and Eq. (8) for turbulent flow

3 Effects of temperature level

The influence of temperature level on the heat transfer is a result of the density variation

of the cooling air with the temperature magnitude, and the embodiment of this variation in the momentum equations is the buoyancy. In turbine disk cavities, many researchers put their focus on the buoyancy produced by centrifugal force in rotating reference frame, without considering interactions of other forces with cooling air density in the governing equation. However, the integral effect of temperature level on density of the cooling air, subsequently on heat transfer of the free disk, would be investigated in this section. For laminar flow, ZHAO et al. [16] provided detailed discussion about the effects of temperature level on heat transfer on the free disk, and some of their works has been also quoted here to compare with results for turbulent flow.

Local Nusselt number of the free disk with compressible and incompressible cooling air were calculated and compared to obtain the influence of temperature level on heat transfer. The relative difference ϵ_r is defined as below:

$$\epsilon_r = \frac{Nu_{r,incp} - Nu_{r,cp}}{Nu_{r,incp}} \quad (9)$$

where $Nu_{r,cp}$ is the local Nusselt number computed for the compressible cooling air and $Nu_{r,incp}$ for the incompressible cooling air with same numerical conditions. On the surface of the free disk, monomial temperature profiles of Eq. (5) were applied. To scale the level of the disk temperature, the nondimensional max temperature difference θ and nondimensional local temperature difference θ_r are defined as below:

$$\theta = \frac{T_{w,R} - T_f}{T^*} = \frac{Cr^n}{T^*} \quad (10)$$

$$\theta_r = \frac{T_{w,r} - T_f}{T^*} = \frac{T_w - T_f}{T^*} = \frac{Cr^n}{T^*} \quad (11)$$

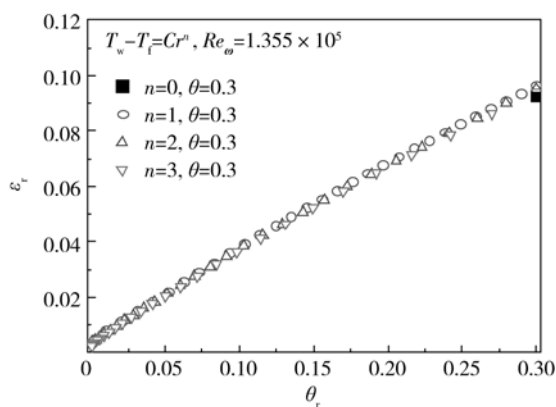
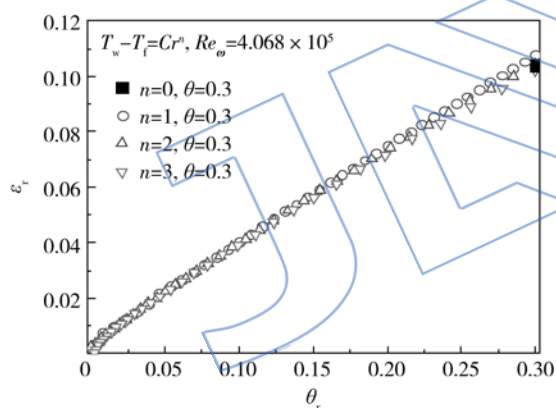
where $T^* = 300$ K is a preference temperature chosen to scale the disk temperature, and it is not related to the opening boundary temperature T_f . For a specific n in Eq. (10), the coefficient C is determined by θ , and Table 1 shows the values of C and θ used in the numerical simulation.

3.1 Variation of the relative difference with nondimensional local temperature difference

Fig. 10 and Fig. 11 demonstrate the effect of the

Table 1 Values of coefficient C and dimensionless maximal temperature difference θ

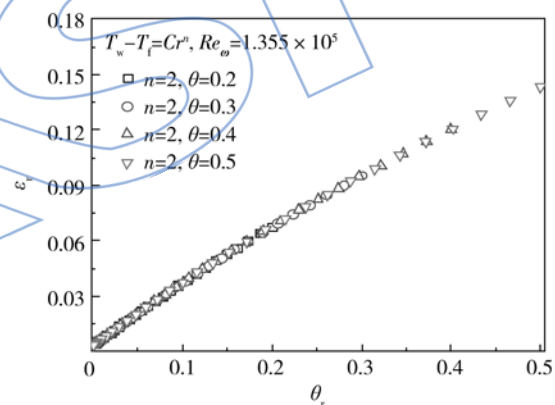
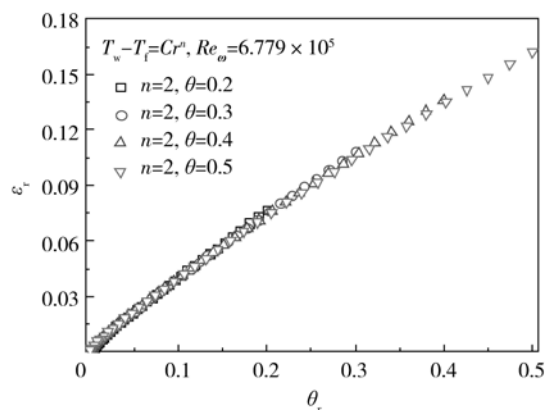
n	$C(\theta)$			
0	60 (0.2)	90 (0.3)	120 (0.4)	150 (0.5)
1	300 (0.2)	450 (0.3)	600 (0.4)	750 (0.5)
2	1500 (0.2)	2250 (0.3)	3000 (0.4)	3750 (0.5)
3	7500 (0.2)	11250 (0.3)	15000 (0.4)	18750 (0.5)

Fig. 10 Effect of n on variation of ϵ_r with θ_r for laminar flowFig. 11 Effect of n on variation of ϵ_r with θ_r for turbulent flow

exponent n on the relationship between the relative difference ϵ_r and the nondimensional local temperature difference θ_r for laminar and turbulent flow respectively. With the increasing of θ_r , ϵ_r rise and this phenomenon indicates that when the temperature level increases, the heat transfer capacity of compressible cooling air is weakened comparing with the incompressible cooling air. Both for the two flow state, the influence of n on the variation of ϵ_r with θ_r is quite weak, for example, the max deviation of ϵ_r for different n

is about 3% for laminar flow and 5% for turbulent flow, so it is reasonable to neglect this effect.

Fig. 12 and Fig. 13 show the influence of the nondimensional max temperature difference θ on the variation of ϵ_r with θ_r for laminar and turbulent flow respectively. It is easy to see that with appointed exponent n and rotating Reynolds number Re_ω , ϵ_r is determined by θ_r and is independent of θ almost.

Fig. 12 Effect of θ on variation of ϵ_r with θ_r for laminar flowFig. 13 Effect of θ on variation of ϵ_r with θ_r for turbulent flow

Considering the effect of rotating Reynolds number Re_ω on the relationship between ϵ_r and θ_r , Fig. 14 shows the results of laminar flow. When appointed n and θ , the deviation of the value of ϵ_r for different Re_ω is very small. For turbulent flow, as shown in Fig. 15, when θ_r becomes bigger, at high local radius in situ, the effects of Reynolds number on ϵ_r is gradually clear, especially for larger Re_ω . The max difference of ϵ_r for the given values of Re_ω is about 11%. As the assumption of idea gas is applied and the turbulent case has relatively higher Reynolds number, the pressure and density of the cooling air near the edge of disk is declined obviously with increasing Re_ω (see Dorfman^[8] for detail), and subsequently, the heat transfer capacity of the cooling air is also weakened. However, it is rational to neglect the influences of Re_ω on ϵ_r , as the main intention of this paper is

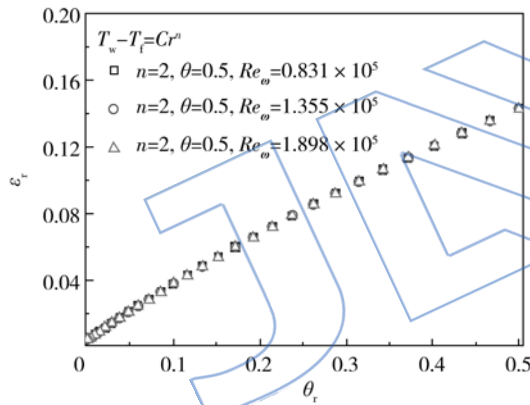


Fig. 14 Effect of Re_ω on variation of ϵ_r with θ_r for laminar flow

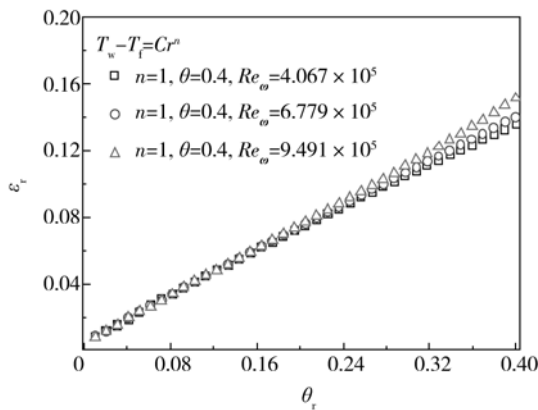


Fig. 15 Effect of Re_ω on variation of ϵ_r with θ_r for turbulence

to investigate effects of temperature boundary condition on heat transfer on the free disk and the range of Re_ω applied in the CFD simulation is relatively small.

3.2 Numerical correlation formulas

Observing the effects of temperature level on heat transfer separately, results in section 3.1 reveal that the local heat transfer on the free disk for compressible flow is dominantly controlled by local temperature level, i. e., ϵ_r is determined by θ_r and is not related to n , θ and Re_ω both for laminar and turbulent flow. So for the free disk, it is reasonable to assume the following function that

$$\epsilon_r = f(\theta_r) \quad (12)$$

By fitting all the numerical data, correlation formula of the relative difference ϵ_r and the nondimensional local temperature difference θ_r is obtained for laminar and turbulent flow respectively as follows.

For the laminar flow:

$$\epsilon_r = 0.25\theta_r^{0.815} \quad (13)$$

The computed range is: $0 \leq Re_\omega \leq 2 \times 10^5$, $0 \leq n \leq 3$ and $0 \leq \theta \leq 0.5$. Fig. 16 is the diagram of data fitting, and there are more than 99.2% points located in the confidence interval of $\pm 3\%$.

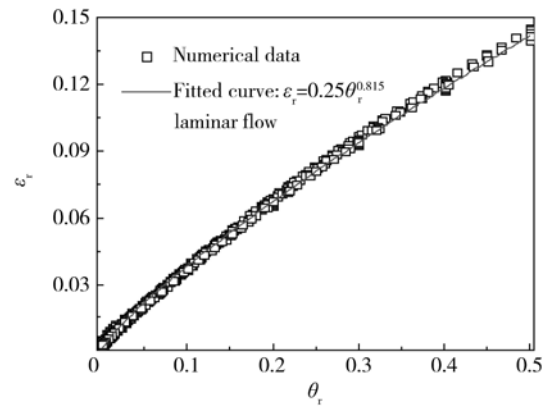


Fig. 16 Fitted curve of ϵ_r versus θ_r for laminar flow

For the turbulent flow:

$$\epsilon_r = 0.295\theta_r^{0.86} \quad (14)$$

The computed range is: $2 \times 10^5 \leq Re_\omega \leq 2 \times 10^6$, $0 \leq n \leq 3$ and $0 \leq \theta \leq 0.5$. As shown in Fig. 17 is

the fitted curve, and more than 99% points located in the confidence interval of $\pm 7\%$.

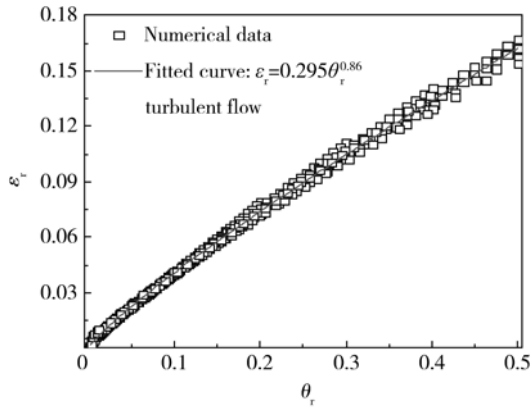


Fig. 17 Fitted curve of ϵ_r versus θ_r for turbulent flow

4 Correction of local Nusselt number

4.1 Correction method

Through the analysis mentioned above, it draws an understanding of the influence of temperature boundary condition on heat transfer on a free disk, and this review may provide a meaningful reference for the engineering. In aero-engines, the temperature profile on disks of turbines or compressors is rather complicated and high, so the heat transfer measurement on disks is usually conducted with simple flow and thermal boundary conditions in practice, such as ordinary and low temperature profiles are used on the disk surface. Consequently, it is significant to seek a correction method to revise the experimental data.

If the local Nusselt number on a experimental disk with uniform and low temperature is measured, say $Nu_{r,exp}$, then the values of local Nusselt number on a real turbine or compressor disk with complicated and high temperature, such as $Nu_{r,real}$, may be determined by revised $Nu_{r,exp}$. A potential method of correcting the local Nusselt number on the turbine disk from $Nu_{r,exp}$ to $Nu_{r,real}$ is listed as follows.

Step 1: Calculate the local Nusselt number, say $Nu_{r,step1}$, for the arbitrary distribution of temperature. Firstly, fit the temperature data

on the real disk to an n th order polynomial as Eq. (6); secondly, in the monomial temperature profile case as Eq. (5), estimate the values of the ratio of local Nusselt number $\sigma_{r,n}$ for $n=1,2,\dots,n$, defined in Eq. (7), using theoretical, numerical or experiential results; finally, compute the values of $Nu_{r,step1}$ using Eq. (8) with substitution of $Nu_{r,0} \approx Nu_{r,exp}$, then

$$Nu_{r,step1} = Nu_{r,0} \left(\frac{a_0 + a_1 r \sigma_{r,1} + a_2 r^2 \sigma_{r,2} + \dots + a_n r^n \sigma_{r,n}}{a_0 + a_1 r + a_2 r^2 + \dots + a_n r^n} \right) \approx Nu_{r,exp} \left(\frac{a_0 + a_1 r \sigma_{r,1} + a_2 r^2 \sigma_{r,2} + \dots + a_n r^n \sigma_{r,n}}{a_0 + a_1 r + a_2 r^2 + \dots + a_n r^n} \right) \quad (15)$$

Step 2: Calculate the local Nusselt number, say $Nu_{r,step2}$, based on the values of $Nu_{r,step1}$ from step 1, with considering the effects of temperature level. Replace Eq. (9) by Eq. (12), then

$$Nu_{r,step2} = Nu_{r,step1} - \epsilon_r Nu_{r,step1} = Nu_{r,step1} - f(\theta_r) Nu_{r,step1} \quad (16)$$

where $Nu_{r,step2}$ is also the corrected local Nusselt number, i. e. $Nu_{r,step2} \approx Nu_{r,real}$. For the free disk, $f(\theta_r)$ is determined by Eq. (13) for laminar flow and Eq. (14) for turbulent flow.

4.2 Validating of the correction method

To check the validity of the correction method, numerical simulations were performed for the free disk, and the numerical condition is the same as mentioned in section 1. The local Nusselt number calculated in uniform temperature profile for incompressible air was used to model $Nu_{r,exp}$, and calculated in given polynomial temperature profile for compressible air was used to simulate $Nu_{r,real}$.

Fig. 18 and Fig. 19 illustrate the local Nusselt numbers of $Nu_{r,exp}$, $Nu_{r,real}$, $Nu_{r,step1}$ and $Nu_{r,step2}$ versus nondimensional radius r/R of the free disk for laminar and turbulent flow respectively. As shown in these charts, with the given specific temperature boundary conditions, $Nu_{r,exp}$ is less than $Nu_{r,real}$ in the max value by 9.3% for laminar flow and 5% for turbulence. $Nu_{r,exp}$ seems to be acceptable in some engineering practices, but this is just for this tempera-

ture profile, and if temperature applied on the disk has big magnitude or large gradient along the flow direction separately, the phenomenon may be changed. Through the correction of step 1, without considering the effect of temperature level, $Nu_{r, \text{step 1}}$ departs from $Nu_{r, \text{real}}$ obviously. However, after the 2-step correction $Nu_{r, \text{step 2}}$ is consistent with $Nu_{r, \text{real}}$, and their max differences are about $\pm 0.4\%$ both for the two flow states. So the correction method in section 4.1 is reasonable and may be applied to turbine or compressor disks in aero-engines.

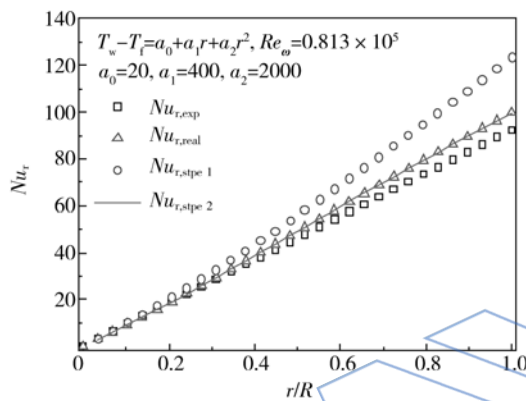


Fig. 18 Revision of local Nusselt number of the free disk for laminar flow

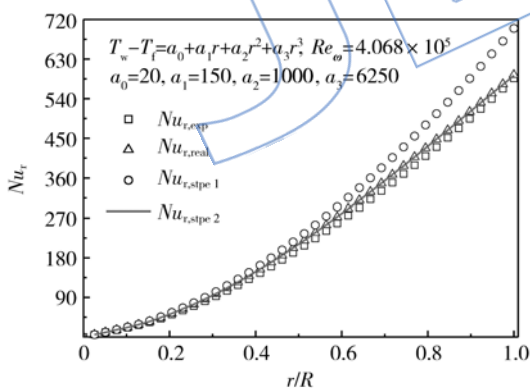


Fig. 19 Revision of local Nusselt number of the free disk for turbulent flow

5 Conclusions

Through the numerical investigation, it presented the effects of the temperature boundary conditions on the heat transfer on a free disk, and similar conclusions can be drawn both for laminar and turbulent flow.

Incompressible cooling air was used separately to observe the variation of local heat transfer with the temperature distribution in the simulation. The results reveal that: with monomial temperature profile and given rotating Reynolds number, the local Nusselt number on the free disk is mainly controlled by the exponent n and is not related to the coefficient C . The theory for the boundary layer flow that the heat transfer coefficient for an arbitrary polynomial temperature profile, with an order n , can be deduced from results of monomial profiles, with orders from 0 to n , is profound on the free disk, and the numerical results obtained were consistent with the theory.

The difference of heat transfer between compressible and incompressible cooling air, mainly caused by the temperature level, was compared by the local Nusselt number on the free disk. The results show that the relative difference of local Nusselt number is mainly affected by nondimensional local temperature difference, and almost independent of other factors. Correlation formulae of the effect of local temperature level on local heat transfer were provided.

Finally, by the means of the simulations and mathematical deductions, a correction method for heat transfer coefficients of the free disk was presented and verified. Therefore, the heat transfer coefficients, obtained by applying uniform and low temperature profiles, can be revised by taking account of any temperature distribution and magnitude.

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